

Stiffness demand for the outer casing in a buckling restrained brace

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A simple model shown in Figure 1 is used to derive the required stiffness of the casing of buckling restrained braces (BRB).

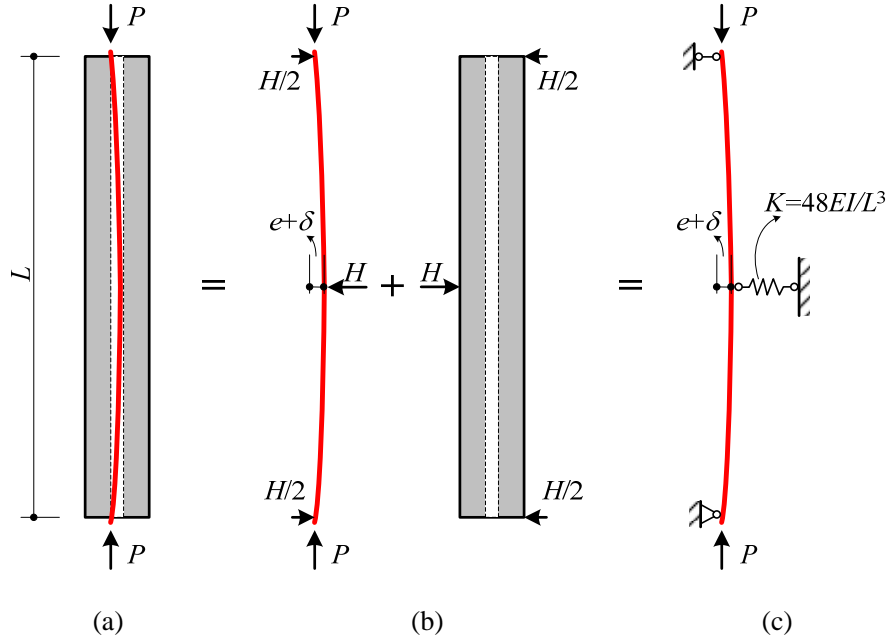


Figure 1. A simple to derive the required stiffness of the casing in a BRB

From the equivalent of moment around the middle point of the core in Figure 1(b), it must have:

$$P(e + \delta) = HL/4 + M_{\text{core}} \approx HL/4 \quad (1)$$

where e is the initial imperfection of the core and δ is the deformation induced by the external compression P ; L is the length of the brace and H denotes the horizontal constraining force provided by the casing; M_{core} is the moment at the middle span section of the steel core, and could be reasonable assumed to be zero, especially when the core has yielded in compression.

On the other hand, the deflection of the casing at the middle of its span would be

$$\delta = H/K = HL^3 / (48EI) \quad (2)$$

where $K=48EI/L^3$ is the bending stiffness of the casing, E and I can be conservatively taken as the Young's modulus and the moment of inertia of only the steel tube.

The horizontal constraining force H can then be worked out by equating the deformation of the core and the casing induced by the compression P :

$$H = \frac{e}{\frac{L}{4P} - \frac{L^3}{48EI}} \quad (3)$$

In order to provide effective constrain to the core from lateral buckling, H should always stay positive. Thus the denominator of the above equation should always stay positive, i.e.:

$$\frac{L}{4P} - \frac{L^3}{48EI} > 0 \quad (4)$$

A criterion of determining the stiffness of the casing can then be obtained by putting the above equation in the following form:

$$\frac{12EI}{L^2} > P_y \quad (5)$$

Here we assume that the maximum compressive force the core can ever sustain is its yield strength P_y . This equation excludes the influence of the degree of imperfection. Instead, another criterion may be drawn by assuming that the moment in the middle of the tube would not exceed its yield moment, M_y , i.e.,

$$M = \frac{LH}{4} = \frac{e}{\frac{1}{P} - \frac{L^2}{12EI}} < M_y = \frac{\sigma_y I}{D} \quad (6)$$

Rearranging [Eq. 6](#) and substitute P by P_y yields

$$\frac{12EI}{L^2} > P_y \left(1 + \frac{12E}{\sigma_y} \cdot \frac{D}{l} \cdot \frac{e}{l} \right) \quad (7)$$

It is similar to [Equation 5](#) but it takes into account the influence of imperfection. With large imperfection, the tube needs to be stronger or stiffer to prevent the development of the global buckling of the core.

On the other hand, it is of the same form as the criterion proposed by [Wada and Nakashima \(2004\)](#) ([Eq. 8](#)).

$$P_e = \frac{\pi^2 EI}{L^2} > P_y \left(1 + \frac{\pi^2}{2} \frac{E}{\sigma_y} \cdot \frac{D}{l} \cdot \frac{e}{l} \right) \quad (8)$$

The coefficients in the left-hand term, i.e., 12 in [Eq. 7](#) and π^2 in [Eq. 8](#), which is different from each other by about 20%, are the consequence of different functions of lateral deformation pattern that are adopted in the derivation. It was assumed in the derivation of [Eq. 8](#) that the lateral deformation of the tube and that of the core follows the same sine function. By contrast, it is assumed for [Eq. 7](#) that the tube is carrying a concentrated load at its middle point and its deflection curve is the result of this concentrated load.

Reference:

Wada, A.; Nakashima, M. (2004). From infancy to maturity of buckling restrained braces research. Proc. 13th World Conference on Earthquake Engineering, Vancouver, B.C., Canada, Paper No. 1732.